# Reading List on Prophet Inequalities

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This list contains an overview of papers covered in the tutorial on Prophet Inequalities at EC'21 and further reading.

## 1 Overview and Applications

- [KS77, KS78] gets the first 2 approx for single item. [SC84] show the median threshold and [KW12] show the half-of-max threshold also get 2 approx. Waggoner [Wag18] shows that any threshold in between the two also works. Motivation to study more general prophet inequalities comes from applications in mechanism design and online algorithms.
- Mechanism design: [HKS07] observed that prophet inequalities can be used to design posted pricing mechanism for welfare maximization. They get $1 + O\left(\frac{\sqrt{\ln k}}{\sqrt{k}}\right)$ approx for selecting $k$ items, which was improved in [Ala14] to a tight $1 + O\left(\frac{1}{\sqrt{k}}\right)$ approx. [CHMS10] first showed that prophet inequalities can also be used for revenue maximization, including multi-parameter settings.
- Online algorithms: Prophet inequality model is useful for Beyond Worst-Case Analysis of Online Algorithms, i.e., problems where we cannot obtain good guarantees in the traditional online model.
- Maximization problems: [CHMS10] introduce for matroids and [KW12] get a tight 2 approximation. For bipartite graphs, [FGL15] get 2 approx in vertex arrival and [FSZ16] get $O(1)$ approx in edge-arrival models (subsequently improved). [Rub16, RS17] get $O(\text{poly} \log n)$ for arbitrary packings.
- Minimization problems: Techniques of [GGLS08] give constant competitive algorithms for online facility location and online steiner tree problem in prophet model, which is not possible in the classic online model. Similarly, [DEH+17] study online $k$ server in the prophet model.

## 2 Algorithms using Posted and Balanced Prices

- Posted Pricing Mechanisms: [FGL15] gets 2 for combinatorial auctions with XOS bidders.
- Balanced-prices: [KW12] gets $4k - 2$ approx for intersection of $k$ matroids. [DFKL17] simplify and generalize this to several other related settings like knapsacks.
- [GW19] gets 3 approx using non-adaptive posted prices for edge-arrival bipartite matching.
- A recent work shows $O(\log \log n)$ for subadditive combinatorial auctions using posted-prices [DKL20].

## 3 Algorithms using Online Contention Resolution Schemes

- [Ala14] gets $1 + O(\frac{1}{\sqrt{k}})$ for selecting $k$ items, without defining OCRS.
- [FSZ16] introduce Online Contention Resolution Schemes and give a 4 approx for matroid prophet inequality. This was later improved to 2 approx in [LS18].
- Edge-arrival bipartite matching $O(1)$ approx in [FSZ16]. Recently, improved to $\approx$ 2.96 approx even for general graphs using OCRS [EFGT20]. For bipartite graphs, this paper [EFGT20] also extends [FGL15] to obtain a 2-approx for two-sided vertex arrivals.

## 4 Getting a PTAS compared to the Optimal Policy

- For single item known adversarial order, a simple DP gives the optimal policy. [ANSS19] gets PTAS for simple laminar constraints in this setting.
- [ASZ20] study free-order single item model. [HFX18] get PTAS in this model. A very recent work improves this result to get an EPTAS [SS20].

## 5 I.I.D., Free-Order, and Prophet Secretary Models

- IID Model: Already studied by Hill and Kertz [HK+82]. Tight bound of $\approx 1.34$ obtained in [CFH+17]. Correa et al. [CDFS19] show you cannot do better than $1/e$ even for iid case (achieved by secretary algorithm) if the distributions are unknown.
- Free-order model: Already studied by Hill [Hil83] who showed adaptivity doesn't help for single item. [CHMS10] consider this model for matroids. [Yan11] gets $e/(e-1)$ for matroids.
- Prophet Secretary model: Introduced in [EHLM17] and get $e/(e-1)$ for single item. This was extended to matroids and combinatorial auctions in [EHKS18] using dynamic pricing algorithms. [AW18, LS18] define and use random order OCRS. A line of work improves $e/(e-1)$ factor for single item [ACK18, BGL+18, CSZ19].

## 6 Other Related Models

- Sample complexity: Introduced in [AKW14] where only sample access to distributions. They define order-oblivious algorithms and give a reduction to the secretary problem. [CDFS19] study iid pophet inequality from samples. [RWW20] improves their result and obtain tight 2 approx result for single-item setting using a single sample from each distribution.
- Robust prophet inequalities [DK19, ISW20].
- Correlations: Negative correlation [RSC+87, RSC91] and linear positive correlation [ISW20].
- Constrained order prophets [ADK21].

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